

Signatures of superfluidity for Feshbach-resonant Fermi gases

J. Kinnunen, M. Rodríguez, and P. Törmä

Department of Physics, NanoScience Center, P.O.Box 35, FIN-40014 University of Jyväskylä, Finland

We consider atomic Fermi gases where Feshbach resonances can be used to continuously tune the system from weak to strong interaction regime, allowing to scan the whole BCS-BEC crossover. We show how a probing field transferring atoms out of the superfluid can be used to detect the onset of the superfluid transition in the high- T_c and BCS regimes. The number of transferred atoms, as a function of the energy given by the probing field, peaks at the gap energy. The shape of the peak is asymmetric due to the single particle excitation gap. Since the excitation gap includes also a pseudogap contribution, the asymmetry alone is not a signature of superfluidity. Incoherent nature of the non-condensed pairs leads to broadening of the peak. The pseudogap and therefore the broadening decay below the critical temperature, causing a drastic increase in the asymmetry. This provides a signature of the transition.

Novel type of superfluid, a Bose-Einstein condensate of molecules consisting of fermionic alkali atoms, has been created recently [1, 2]. Molecules are formed by a Feshbach resonance in the range of parameters where the interaction between the atoms is repulsive. By tuning magnetic fields the resonance can be crossed and attractive interactions realized. Crossing of the resonance can be done adiabatically, in temperatures lower than the critical temperature [3], and fermion pairing investigated [4]. This opens up unprecedented prospects for studying the BCS-BEC crossover from a superfluid of momentum-space pairs to a BEC of real-space dimers. The experiments [1, 3] demonstrate the existence of a superfluid in the BEC limit of the crossover. In this letter we propose a method to observe an explicit signature of the superfluid transition also in the intermediate (high- T_c) and BCS regimes.

Fermionic alkali atoms in the vicinity of a Feshbach resonance are predicted to form superfluids with three components: 1) fermionic Cooper-pairs, 2) molecular bosons formed out of the fermions, 3) non-condensed fermionic pairs corresponding to a pseudogap. Non-condensed pairs and molecules exist also above the critical temperature. The system can be described by the resonance superfluidity theory [5, 6]. In the BEC limit, the superfluid transition is evident like for an atomic BEC, for instance via the bimodal density distribution [1, 3]. In the high- T_c and BCS regimes such a strong effect on the density is not expected. Moreover, strong pseudogap or mean field may be almost indistinguishable from the effects of superfluidity. Several signatures of superfluidity for fermionic atoms have been proposed in [7, 8], however, these works assume simple BCS scenario and do not take into account the pseudogap and the bosonic molecules. Here we discuss a method that can give a clear signature of the resonance superfluidity transition. It is experimentally feasible since a similar technique has already been used to study mean-field effects [9]. The method is based on transferring atoms from the superfluid state into a normal state using electromagnetic fields. There

is an analogy to imposing current over a superconductor-normal metal interface. Instead of a localized tunneling barrier, the transfer is caused by a homogeneous field, enforcing exact momentum conservation. This leads to a resonance-type of response as a function of the applied energy, in contrast to superconductors where the current grows steadily as a function of voltage once the gap energy is exceeded [10].

Interacting gas of two fermionic components can be formed by trapping atoms in two different hyperfine states, let us label the states by $|g\rangle$, $|g'\rangle$. The strength of the interaction between atoms in $|g\rangle$ and $|g'\rangle$ is tuned by magnetic (or optical) fields using a Feshbach resonance. The atoms provide also other hyperfine states (label $|e\rangle$) for which the interatomic scattering lengths can be very different from that between $|g\rangle$ and $|g'\rangle$. This is utilized in [9] to observe mean field effects: atoms are resonantly transferred, say, between $|g\rangle$ and $|e\rangle$, and the difference in the mean fields experienced by $|g\rangle$ and $|e\rangle$ causes an energy shift in the position of the resonance. Similarly, for a superfluid formed of atoms in the $|g\rangle$ and $|g'\rangle$ states, the BCS-gap in the single particle excitation spectrum is predicted to cause a shift in the position of the resonance [7]. The main qualitative difference to the mean-field shift is the asymmetric shape of the peak in the superfluid case. Here we show that a pseudogap can lead to a shifted, asymmetric peak as well. Therefore asymmetry alone is not a signature of superfluidity. The transition, however, is signaled by an abrupt change in the amount of the asymmetry.

The system is described by the Hamiltonian

$$\begin{aligned}
 H = & \sum_{k,\sigma} \varepsilon_k^\sigma c_k^\sigma c_k^\sigma + \sum_q (E_q^0 + \nu) b_q^\dagger b_q \\
 & + \sum_{q,k,k'} U(k,k') c_{q/2+k}^{g\dagger} c_{q/2-k}^{g'\dagger} c_{q/2-k'}^{g'} c_{q/2+k'}^g \\
 & + \sum_{q,k} (g(k) b_q^\dagger c_{q/2-k}^g c_{q/2+k}^{g'} + h.c.) + H_T,
 \end{aligned}$$

$$H_T = \sum_k \frac{\delta}{2} \left(c_k^{\dagger} c_k^e - c_k^{g\dagger} c_k^g - b_k^{\dagger} b_k \right) \\ + \sum_{k,l} \left(M_{kl} c_k^{g\dagger} c_l^e + h.c. \right) + \sum_{klq} \left(D_{qkl} b_q^{\dagger} c_k^{g'} c_l^e + h.c. \right)$$

The operators c_k^{σ} correspond to fermionic atoms in various hyperfine states ($\sigma = \{e, g, g'\}$) and b_q to spinless bosonic molecules formed out of atoms in states $|g\rangle$ and $|g'\rangle$. The fermion and boson energies are $\varepsilon_k = k^2/2m$ and $E_q = q^2/2M$, respectively, with m the fermion mass, $M = 2m$ and $\hbar \equiv 1$. A momentum cutoff is used: $U(k, k') = U \varphi_k \varphi_{k'}$, $g(k) = g \varphi_k$, where $\varphi_k^2 = \exp(-(k/K_c)^2)$. The effect of the Feshbach resonance can be incorporated to a point-like interaction between the atoms, as in H , by using renormalized parameters U , g and ν [6]. The corresponding unrenormalized parameters are: $U_0 = 4\pi a_s/m$ is the fermion-fermion interaction, with a_s the scattering length, g_0 is the boson-fermion coupling, and ν_0 the magnetic field detuning from the Feshbach resonance. Positive ν_0 corresponds to attractive interactions between the atoms, negative ν_0 to repulsive. At the resonance, $\nu_0 = 0$, the system is in the high- T_c region. The limits $\nu_0 \rightarrow \infty$ and $\nu_0 \rightarrow -\infty$ are the extreme BCS and BEC limits, respectively. The parameters used throughout the paper are $g_0 = -5.0 E_F$ and $U_0 = 0.05 E_F$, corresponding to the density of atoms $n = 10^{13} \text{ cm}^{-3}$ and background scattering length $a_s = 176 a_0$. The initial numbers of atoms in states $|g\rangle$ and $|g'\rangle$ are equal, therefore the corresponding chemical potentials $\mu_g = \mu_{g'} \equiv \mu$. The state $|e\rangle$ is assumed initially empty, that is, $\mu_e = 0$.

The observable of interest is the current of atoms from the state $|g\rangle$ to state $|e\rangle$, $I = \langle \dot{N}_e \rangle$. The effect of the electromagnetic field transferring atoms out of the superfluid, i.e. creating the superfluid-normal state interface, is described by H_T . To match the energy difference between the considered hyperfine states, the field can be either an rf-field or laser field creating a Raman transition. In describing the interaction of the field with the molecule (formed of fermions in states $|g\rangle$ and $|g'\rangle$), we have considered only the interaction of the field with the two-level system $|g\rangle - |e\rangle$. This approximation is made having in mind large, extremely weakly bound molecules [1, 3]. We refer to the detuning $\delta = (\omega_e - \omega_g) - \omega_{field}$ by probe field detuning, not to confuse it with the Feshbach detuning ν_0 of the magnetic field.

The particle current $I = \langle \dot{N}_e \rangle = i \langle [H, N_e] \rangle$ is calculated treating H_T as a perturbation and neglecting terms of higher than second order. The solutions for the equilibrium parameters, chemical potential and the excitation gap, are needed for calculating the current. We apply the t -matrix approach to describe the pairing interaction, adopting the procedure used in Ref. [11] for solving the relevant equations: gap, pseudogap and number equations. The chemical potential μ , pseudogap Δ_{pg} and the superfluid order parameter $\tilde{\Delta}_{sc}$ are obtained from the

self-consistent solutions.

Physically, one may interpret the pseudogap as fermion pairs which have a finite net momentum and are formed above the critical temperature (thus the name preformed pairs). At the critical temperature, these pairs condense to the zero net-momentum state. The origin of the pseudogap in the many-body theory are fluctuations of the finite momentum order parameter $\langle c_{-k}^g c_{k+q}^{g'} \rangle$ around the zero-momentum mean $\langle c_{-k}^g c_k^{g'} \rangle$. The zero-momentum mean is the usual BCS order parameter, and has a non-zero value below the transition temperature. For finite momenta q , the mean $\langle c_{-k}^g c_{k+q}^{g'} \rangle$ is always zero. However, it has a non-zero variance, corresponding to fluctuations, also above T_c . This leads to the pseudogap. The t -matrix for the system can be written as a sum of a superfluid term $\tilde{\Delta}_{sc}^2$ containing a pole at $Q = 0$, and a pseudogap contribution that includes the rest of the t -matrix, $\Delta_{pg}^2 = -\sum_{Q \neq 0} t(Q)$ [12]. The total excitation gap is given by $\Delta^2 = \tilde{\Delta}_{sc}^2 + \Delta_{pg}^2$. Pseudogap has a considerable effect in the high- T_c regime but vanishes in the extreme BCS limit.

The pseudogap and the order parameter are qualitatively different and in principle can be distinguished from each other through life-time effects. The incoherent nature of the propagating, non-condensed pairs affects scattering processes and thus incorporates a finite broadening in the self-energy Σ_{pg} . This is taken into account by replacing the self-energy by the model self-energy [13]

$$\Sigma(K) = \frac{\tilde{\Delta}_{sc}^2 \varphi_k^2}{i\omega + \epsilon_k} + \frac{\Delta_{pg}^2 \varphi_k^2}{i\omega + \epsilon_k + i\gamma}, \quad (1)$$

where γ is the finite broadening. Using the model self-energy, we have plotted the fermionic density of states $N(\omega) = -2 \sum_k \text{Im} G(\omega + i0, k)$ in the inset of Fig. 1 for several temperatures. We have chosen the Feshbach detuning $\nu_0 = 2.0 E_F$ (attractive interactions) and the broadening $\gamma = \Delta(T_c) \approx 0.6 E_F$. The plot shows the widening of the peak at $\omega = -\Delta$ as the temperature increases, caused mainly by the emergence of the pseudogap. At lower temperatures, a peak appears also at $\omega = +\Delta$, and in the limit $T \rightarrow 0$ the density of states approaches the BCS limit of two peaks.

The current $I = \langle \dot{N}_e \rangle = i \langle [H, N_e] \rangle$ can now be calculated. Denoting $A(t) = \sum_{kl} M_{kl} c_k^{g\dagger} c_l^e$ and $B(t) = \sum_{qkl} D_{qkl} b_q^{\dagger} c_k^{g'} c_l^e$, where M_{kl} , D_{qkl} are the coupling constants, the correlations of the form $\langle AA \rangle$, $\langle BB \rangle$, $\langle BA \rangle$ and $\langle B^\dagger A \rangle$ (and $h.c.$) are neglected. The first three contain correlations of the form $\langle c^e c^e \rangle$ which are identically zero for a normal state. The last one corresponds to a Josephson current between the molecular and fermion superfluids, mediated by the $|e\rangle$ state [14].

Writing the total current as a sum of 'fermionic' and 'bosonic' currents $I = I_F + I_B$, one obtains using Matsubara Green's functions techniques

$$\begin{aligned}
I_F &= -2 \sum_{kl} |M_{kl}|^2 \text{Im} \left\{ \sum_{x_n^g} n_F(x_n^g) G_e^{\text{ret}}(m, x_n^g - \delta_F) \text{Res}_{z=x_n^g} G_g(n, z) + n_F(\epsilon_m^e) G_g^{\text{adv}}(n, \epsilon_m^e + \delta_F) \right\} \\
I_B &= \frac{2\pi}{a_0} \sum_{qkl} |D_{qkl}|^2 [n_F(\epsilon_k^e) + n_B(q^2 a_1)] [n_F(E_l + \delta_B) - n_F(E_l)] \frac{E_l + \epsilon_l^g}{2E_l} \delta(E_l + \epsilon_k^e - q^2 a_1 + \delta_B), \quad (2)
\end{aligned}$$

where x_n^g are the poles of the dressed Green's function $G_g(n, z) = (z - \epsilon_n^g - \Sigma(n, z))^{-1}$ and the coefficients a_0 and a_1 are obtained from the linearisation of the boson propagator $D(Q)^{-1} \approx a_0(i\omega - q^2 a_1)$. The self-energy Σ receives an imaginary part from the broadening γ , giving also a nonvanishing imaginary part to the poles x_n^g . For $\gamma = 0$, the fermionic current I_F simplifies to the standard Fermi Golden rule form. We have neglected the backward process in the boson term I_B as the $|e\rangle$ -states are initially unoccupied. The detunings are $\delta_F = \delta_B = -\mu + \delta - \Delta E_{\text{mf}}$, where $\Delta E_{\text{mf}} = E_{\text{mf}}^e - E_{\text{mf}}^g$ is the difference in the mean field energies of the states $|g\rangle$ and $|e\rangle$.

The current I as a function of the probe field detuning δ is plotted in Fig. 1. The chosen Feshbach detuning $\nu_0 = 2.0 E_F$ corresponds to the BCS side of the resonance (in the high- T_c regime, $T_c \approx 0.27 E_F$). Due to small number of molecules on the BCS side, we use $I = I_F + I_B \approx I_F$. For comparison, we plot also the current neglecting the pseudogap ($\Delta_{pg} = 0$) in the lower figure. The main observations from the results are: 1) *Effect of mean field interactions* is simply a shift of the resonance peak by ΔE_{mf} . This is shown by the symmetric Lorentzian at the center (solid line). It corresponds to a high temperature ($\sim T_F$) with no preformed pairs. (Comparison: for $\Delta_{pg} = 0$, the simple mean field behaviour persists until the critical temperature, the solid line Lorentzian is for $T = T_c$.) 2) *Effect of pairing and excitation gap* appears as asymmetric form and splitting of the peak. Pair formation occurs already above the critical temperature, the dashed line for $T = T_c$ represents also $T \gtrsim T_c$. Splitting of the peak is due to opening of an excitation gap. The side-peak on the left corresponds to breaking a pair: the gap energy Δ has to be provided, causing a shift from the center. The side-peak on the right originates from finite temperature quasiparticles, which are excitations with the minimum energy Δ . Energy is *gained* when transferring them from $|g\rangle$ to $|e\rangle$, therefore this side-peak appears on the opposite side of the center position. The quasiparticle side-peak is weaker and vanishes with decreasing temperature, leading to an asymmetric form of the total curve. An important feature is that the curve is broadened by the finite lifetime of the pseudogap. (Comparison: for $\Delta_{pg} = 0$, pair formation occurs only below T_c . Moreover, the pseudogap lifetime broadening is absent which leads to sharper features,

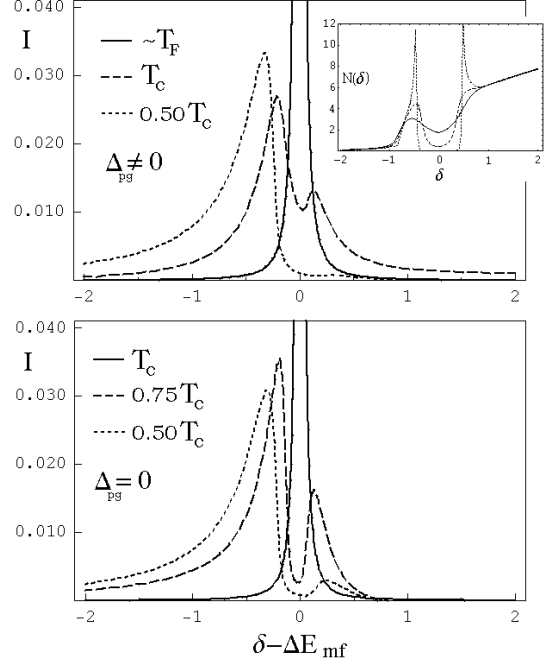


Figure 1: The particle current as a function of the probe field detuning δ for different temperatures. The decay rate of the non-condensed pairs is chosen $\gamma = \Delta(T_c) \approx 0.6 E_F$. For comparison, the lower plot shows the current neglecting the pseudogap. The inset shows the density of states of fermions at resonance detuning $\nu_0 = 2.0 E_F$. There are two peaks located at $\omega = \pm \Delta$, but increasing temperature broadens the peaks. The three plots are for temperatures $T = T_c$ (solid), $0.75 T_c$ (dashed) and $0.10 T_c$ (dotted).

e.g. the two side-peaks are more visible. For $\Delta_{pg} = 0$, the existence of the excitation gap equals superfluidity.) 3) *Superfluidity* is signaled by increase in the asymmetry of the peak (c.f. the dotted curve at $T = 0.5 T_c$) as will be discussed in the following.

We quantify the asymmetry of the peak $A \equiv \frac{v_{\text{left}}}{v_{\text{right}}}$ by integrating the current I to the left v_{left} and to the right v_{right} from the peak value. The asymmetry has been plotted as a function of temperature in Fig. 2, signaling the phase transition by an abrupt change in the asymmetry as a function of temperature. The data for temperatures above T_c have been obtained by extrapolating the total excitation gap $\Delta(T)$ and solving the chemical potential μ from the number equation. Since the magnitude of

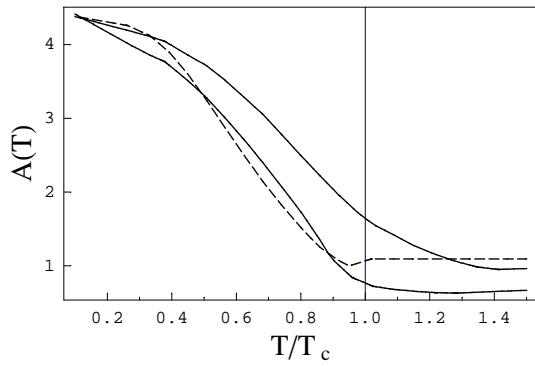


Figure 2: The asymmetry A of the current I as a function of temperature T/T_c for the Feshbach detuning $\nu_0 = 2.0 E_F$ (attractive interaction). Above critical temperature (in the pseudogap regime) the asymmetry is essentially constant but rapidly increases in the superfluid regime. The lower solid line is for lifetime broadening $\gamma \approx \Delta(T = T_c)$ and the upper line for $\gamma \approx 0.2 \cdot \Delta(T = T_c)$. For larger γ , the pseudogap pairs decay faster and are easier to distinguish from Cooper pairs, the lower line therefore shows sharper features at T_c . The dashed line neglects the pseudogap, showing the BCS-like behaviour. In the limit $T \rightarrow 0$, the pseudogap vanishes and all three curves approach the same limit. Averaging with relative densities $n = 1, 0.8$ and 0.6 is included.

the lifetime broadening γ is important for the effectiveness of the method, we show the asymmetry for two γ 's of different orders of magnitude; $\gamma \sim \Delta(T = T_c)$ and $\gamma \sim 0.2\Delta(T = T_c)$ in Fig. 2. Time-energy uncertainty relation suggests $\gamma = t_{\text{decay}}^{-1} \sim \Delta$ which is also used in [15]. Note that the change in the asymmetry at $T = T_c$ is visible also for the smaller γ .

The main experimental challenge related to the method is likely to be mean field effects. Figure 1 presents results for a homogeneous system where the mean field causes merely a shift of the resonance position. In real experiments, the gas is trapped in a harmonic potential and consequently the mean field shift varies depending on the local density in the trap. This leads to broadening of the shifted resonance peak [9]. In order to estimate trapping effects, in the sense of local density approximation, we have calculated the current and the asymmetry for relative densities $n = 1.0, 0.8$ and 0.6 and mean fields $\Delta E_{\text{mf}}(n) = 0.5 E_F$. Fig. 2 presents averaged results: the change in the asymmetry is nevertheless clear. The choice of the state $|e\rangle$ in such a way that mean field effects are under control, possibly combined with schemes that probe only the center of the trap, will be crucial for the success of the method.

Further challenges appear when going further towards the BCS limit. The obscuring effect of the pseudogap and the useful (for this method) effect of the lifetime broadening decrease hand in hand, thus in principle there is no essential difference to the scenario presented here [16]. However, in the extreme BCS regime only atoms

close to the Fermi surface are paired and the atoms far from it will be transferred resonantly without the shift of the peak by the gap energy. In this case, Pauli blocking the lowest energy states for atoms $|e\rangle$ by having them initially populated would strengthen the signature.

In summary, we have studied the possibility of observing resonance superfluidity by a probing field transferring atoms out of the superfluid. The emergence of the pseudogap obscures the phase transition signature because it makes the transfer resonance peak asymmetric even above T_c . However, the incoherent nature of the pseudogap broadens the quasiparticle peaks in the spectral densities, allowing one to distinguish between the pseudogap and superfluid regimes.

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current I_B is not negligible.